POST-GRADUATE COURSE

Term End Examination — June, 2022/December, 2022

MATHEMATICS

Paper-9A(ii) : OPERATIONS RESEARCH

(Applied Mathematics)

(Spl. Paper)

Time : 2 hours] [Full Marks : 50

Weightage of Marks : 80%

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The marks for each question has been indicated in the margin.

Use of scientific calculator is strictly prohibited.

Answer Question No. **1** and any *four* from the rest :

1. Answer any *five* questions : $2 \times 5 = 10$

a) Using Newton's method find Min $f(x, y) = x - y + 2x^2 + 2xy + 2y^2$ with $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as starting point.

b) Define real valued unimodal function with an example.

c) If $\{F_n\}$ be the Fibonacci sequence then find the value of $\sum_{n=0}^{10} F_n$.

d) What is the Golden ratio ? Find the value of the Golden ratio.

e) State convergence criteria for unconstrained gradient-based optimization method.

f) Write three applications of Dual Simplex method.

g) When the Kuhn-Tucker necessary condition is also be sufficient condition ?

2. a) Write down the advantages of Revised Simplex method.

b) Solve the following problem by using Kuhn-Tucker conditions :

Maximize $Z = 5 + 8x_1 + 12x_2 - 4x_1^2 - 4x_2^2 - 4x_3^2$

Subject to $x_1 + x_2 \leq 1$

$2x_1 + 3x_2 \leq 6$

$x_1, \; x_2 \geq 0$ 3 + 7

3. a) Describe Gomory's Cutting Plane Algorithm to solve integer programming problem.

b) Solve the following Integer Programming Problem by using Branch and Bound Algorithm :

Maximize $Z = 4x + 3y$

Subject to $3x + 4y \leq 12$

$4x + 2y \leq 9$

$x, y \geq 0$ and x, y are integers. 4 + 6

4. a) Write down Beale's algorithm for solving quadratic programming problem (QPP).

   b) Using Beale's method solve the following QPP :

   Maximize $Z = 5 + 4x + 6y - 2x^2 - 2xy - 2y^2$

   Subject to $x + 2y \leq 2,\ x, y \geq 0$. 4 + 6

5. a) If $f(x)$ is quadratic then prove that the minimum point can be obtained in a single step by Newton's method.

   b) Minimize $f(x) = |x - 1|$ in the interval $[-1, 5]$ by Fibonacci method using n = 5 and find the final interval of uncertainty. 4 + 6

6. a) In post optimality analysis, how does the optimal solution change due to discrete change in the cost vector ?

   b) The optimal solution of the L. P. P.

   Maximize $Z = 6x_1 - 2x_2 + 3x_3$

   Subject to $2x_1 - x_2 + 2x_3 \leq 2$

   $x_1 + 4x_3 \leq 4$

   $x_1, x_2, x_3 \geq 0$

   is contained in the following table :

C_B	B	X_B	a_1	a_2	a_3	a_4	a_5
6	a_1	4	1	0	4	0	1
– 2	a_2	6	0	1	6	– 1	2
$Z_j - C_j$		$z = 12$	0	0	9	2	2

   Find the range of cost components when changed all three at a time to keep the optimal solution same. 5 + 5

7. a) Using Davidon-Fletcher-Powell method

   Minimize $f(x_1, x_2) = 2x_1^2 + 4x_2^2 - 12x_1 + 16x_2 + 41$ with $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as starting point.

   b) Prove that the function $f(x)$ increases at the faster rate in the direction of $\vec{\nabla} f$. 7 + 3